STAT 131 — Discussion (Lecture 8) Solutions

Prepared for students
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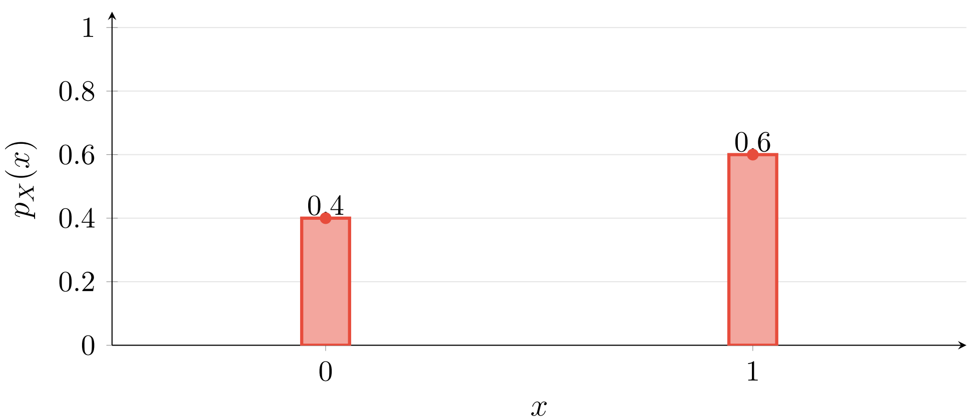
Reading the figures

- **PMF (bar plot):** heights are probabilities; they sum to 1.
- **CDF (step plot, right–continuous):** jumps occur exactly at points with positive probability; dashed line at $y = 0.5$ helps identify a median.
- **Discrete median test:** A number m is a median if $P(X \le m) \ge \frac{1}{2}$ and $P(X \ge m) \ge \frac{1}{2}$.

1. PMF and CDF for $X \sim \text{Bernoulli}(0.6)$

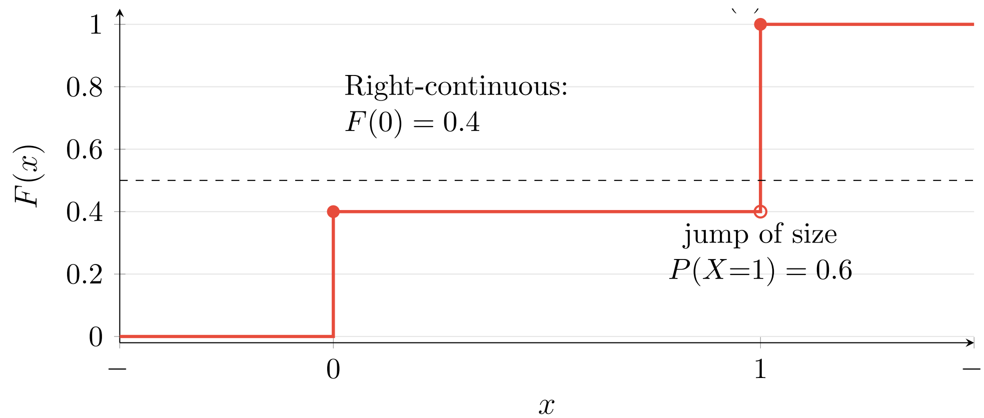
Basics. $P(X = 1) = 0.6,\ P(X = 0) = 0.4.$

(a) PMF (bar chart)



Note. $0.4 + 0.6 = 1$. The support is $\{0, 1\}$.

(b) CDF (step plot)



This shows the jump sizes equal the point masses: $F(0) - F(0^-) = 0.4$ at $x = 0$, and $F(1) - F(1^-) = 0.6$ at $x = 1$.

2. Medians via the CDF

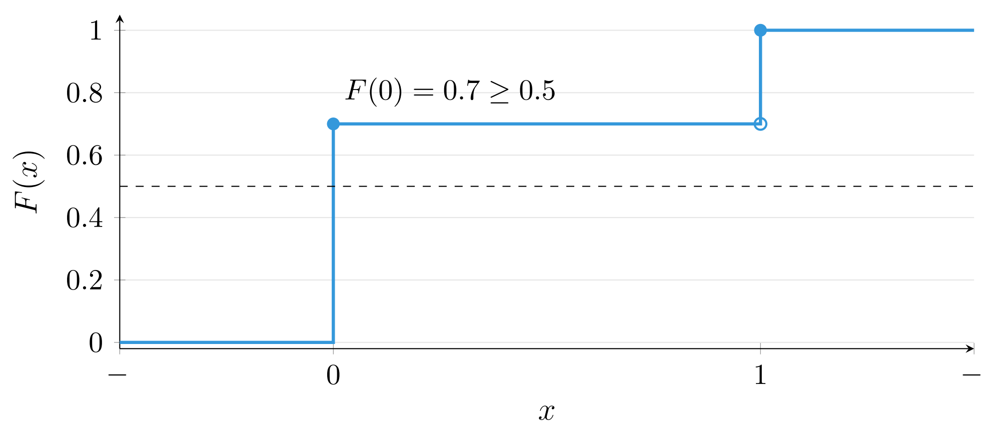
(a) $X \sim \text{Bernoulli}(0.6)$

$F(0) = 0.4 < \frac{1}{2}, \quad F(1) = 1 \geq \frac{1}{2}$. Also $P(X \geq 1) = 0.6 \geq \frac{1}{2}$ while $P(X \geq 0) = 1$. Hence the discrete median criterion holds only for $m = 1$.

$$\boxed{\text{Median}(X) = 1.}$$

(b) $X \sim \text{Bernoulli}(0.3)$ (CDF for clarity)

Here $P(X{=}1) = 0.3, \; P(X{=}0) = 0.7$.



Since $F(0) = 0.7 \geq \frac{1}{2}$ and $P(X \geq 0) = 1$, $m = 0$ is a median. But $P(X \geq 1) = 0.3 < \frac{1}{2}$, so $m = 1$ is not.

$$\boxed{\text{Median}(X) = 0.}$$