STAT 131 — Discussion - Lecture 5: Solutions

Prepared for students

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How to read these solutions

We first **set up** the random variables/events, then give the **key idea** (counting or a formula), and **box** the final answers. Short reminders explain why each step works.

Problem 1. Two balanced 4-sided dice: X (sum) and Y (absolute difference)

Let the dice be (D_1, D_2) with outcomes in $\{1, 2, 3, 4\}$; all 16 ordered pairs are equally likely.

(a) pmf of $X = D_1 + D_2$

Possible sums: $2, 3, 4, 5, 6, 7, 8$. Counts across the 4×4 table grow then shrink ("triangular" pattern).

$$\#\{(D_1, D_2) : D_1 + D_2 = x\} = \min(x - 1,\ 9 - x), \qquad x = 2, \ldots, 8.$$

Thus

$\mathbb{P}(X = 2) = \frac{1}{16}$, $\mathbb{P}(X = 3) = \frac{2}{16}$, $\mathbb{P}(X = 4) = \frac{3}{16}$, $\mathbb{P}(X = 5) = \frac{4}{16}$, $\mathbb{P}(X = 6) = \frac{3}{16}$, $\mathbb{P}(X = 7) = \frac{2}{16}$, $\mathbb{P}(X = 8$

$$\text{Compactly:} \quad \boxed{\mathbb{P}(X = x) = \frac{\min(x - 1,\ 9 - x)}{16},\ \ x = 2, \ldots, 8.}$$

(b) pmf of $Y = |D_1 - D_2|$ and most likely Y

Possible values: $0, 1, 2, 3$. Count by "offset" y:

$\#\{(i, j) : |i - j| = 0\} = 4, \quad \#\{(i, j) : |i - j| = 1\} = 6, \quad \#\{(i, j) : |i - j| = 2\} = 4, \quad \#\{(i, j) : |i - j| = 3\} =$

Hence

$$\mathbb{P}(Y = 0) = \tfrac{4}{16} = \tfrac{1}{4}, \quad \mathbb{P}(Y = 1) = \tfrac{6}{16} = \tfrac{3}{8}, \quad \mathbb{P}(Y = 2) = \tfrac{4}{16} = \tfrac{1}{4}, \quad \mathbb{P}(Y = 3) = \tfrac{2}{16} = \tfrac{1}{8}.$$

$$\text{Most likely value of } Y : \boxed{1} \quad \text{(probability } 3/8).$$

(c) Sketching the pmfs (what the bar plots look like)

- For X: bars over $x = 2, \ldots, 8$ with heights $1, 2, 3, 4, 3, 2, 1$ divided by 16 (a symmetric triangle peaked at $x = 5$).
- For Y: bars over $y = 0, 1, 2, 3$ with heights 1/4, 3/8, 1/4, 1/8 (peak at $y = 1$).

Problem 2. Overbooking with no-shows

Each of the 52 ticketed passengers independently shows up with probability $p = 0.95$ (no-show$= 0.05$). Let $N \sim \text{Binomial}(n = 52, p = 0.95)$ be the number who show up. The plane has 50 seats, so we need

$$\mathbb{P}(\text{enough seats}) = \mathbb{P}(N \leq 50) = 1 - \mathbb{P}(N \geq 51) = 1 - \big[\mathbb{P}(N = 51) + \mathbb{P}(N = 52)\big].$$

Compute the two terms:

$$\mathbb{P}(N = 51) = \binom{52}{51}(0.95)^{51}(0.05), \qquad \mathbb{P}(N = 52) = (0.95)^{52}.$$

Therefore

$$\boxed{\mathbb{P}(N \leq 50) = 1 - \binom{52}{51}(0.95)^{51}(0.05) - (0.95)^{52}}.$$

Numerical value. Plugging in gives approximately

$$\boxed{\mathbb{P}(N \leq 50) \;\approx\; 0.741} \qquad \text{(about a 74.1\% chance of having enough seats).}$$

Counting tip: for sums of dice, read the 4×4 table along diagonals; for absolute differences, count by offsets. For binomial tails, use the complement to keep the sum short.