STAT 131 — Discussion Solutions (Lec 3 & Lec 4)

Prepared by Antonio Aguirre
University of California, Santa Cruz

How to read these solutions

For each item we **set up** the notation, give the **key idea** in one line, and **box** the final result. Concise, rigorous, and student–friendly.

Discussion (Lec 3)

1. Substrings of length 7 from a string of length 10

A substring is *contiguous*. Choose the start index $s \in \{1, 2, \ldots, 10 - 7 + 1\} = \{1, 2, 3, 4\}$.

$$\text{Number of substrings} = \boxed{10 - 7 + 1 = 4}.$$

2. Randomly hiring 3 students from STAT 131

Assume N students in the class, and 3 are chosen uniformly at random without replacement.

$$\Pr(\text{“you” are selected}) = \frac{\binom{N-1}{2}}{\binom{N}{3}} = \boxed{\frac{3}{N}}.$$

Example: If $N = 116$, then $\Pr = \boxed{\frac{3}{116} \approx 0.0259}$.

Discussion (Lec 4)

Setup

Each child independently has curly hair with probability $p = \frac{1}{4}$. Let $X \sim \text{Binomial}(5, p)$ be the total number of curly–haired children among five.

1(a). Given “at least one curly”, find $P(X \geq 3 \mid X \geq 1)$

$$P(X \geq 3 \mid X \geq 1) = \frac{P(X \geq 3)}{P(X \geq 1)} = \frac{1 - P(X \leq 2)}{1 - P(X = 0)}.$$

Compute (with $p = \frac{1}{4}$, $q = \frac{3}{4}$):

$$P(X = 0) = q^5 = \tfrac{243}{1024},$$
$$P(X = 1) = \binom{5}{1} pq^4 = \tfrac{405}{1024},$$
$$P(X = 2) = \binom{5}{2} p^2 q^3 = \tfrac{270}{1024}.$$

Hence $P(X \geq 3) = 1 - \frac{918}{1024} = \frac{53}{512}$ and $P(X \geq 1) = 1 - \frac{243}{1024} = \frac{781}{1024}$, so

$$P(X \geq 3 \mid X \geq 1) = \frac{53/512}{781/1024} = \boxed{\frac{106}{781} \approx 0.1357}.$$

1(b). Given "the youngest has curly hair", find $P(X \geq 3 \mid \text{youngest curly})$

Conditioning on the youngest being curly fixes one success; among the remaining 4 children, let $Z \sim \text{Binomial}(4, p)$. We need $P(Z \geq 2)$:

$$P(Z = 0) = q^4 = \tfrac{81}{256}, \qquad P(Z = 1) = \binom{4}{1} pq^3 = \tfrac{108}{256},$$

$$P(Z \geq 2) = 1 - (P(Z = 0) + P(Z = 1)) = 1 - \tfrac{189}{256} = \boxed{\tfrac{67}{256} \approx 0.2617}.$$

1(c). Why (a) and (b) differ

"At least one curly" is weak information: it merely rules out $X = 0$. In (b) we know a *specific child* is curly, which effectively reduces the problem to getting at least 2 successes out of 4 additional trials, making the event $X \geq 3$ more likely. Formally, (b) conditions on a smaller, more informative event, hence the higher probability.

Tip: for binomial conditionals, rewrite the event after conditioning (e.g., "need at least 2 of the remaining 4").